Approaches to Analytical Solutions of Laminar Compressed Free Jets

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Abstract

WE study here the effect of a uniform mass density time variation on a subsonic gas jet by generalizing the classical similarity hypothesis. Both plane and circular laminar jet evolutions are precisely established and compared.

I. Laminar Jet Self-Preserving Region

We are first interested in characterizing the behavior of the plane or axisymmetric laminar jet self-preserving region. The jet flow is well into the subsonic flow so that the gas mass density ρ is uniform in space and governed by the displacement of the pistons (see Fig. 1).

Two characteristic times rule the flow evolution – the mass density variation time scale

$$t_\rho = \frac{\max|\rho(t)|}{\max|d\rho/dt|}$$

and the jet characteristic time scale (see Fig. 1)[1]:

$$t_j = \frac{L_j}{U_{\text{axe}}} = \frac{R_e\, l_j}{U_{\text{axe}}}$$

The nonstationary boundary-layer equations are still valid when $t_p = o\ (t_j)$ (nonstationary evolution) and $t_p \gg t_j$ (pseudostationary evolution).

In these two cases, we intend to generalize the affinity property of the stationary jet velocity profile[2] by searching for a solution of the following type:

$$\frac{U(x, y, t)}{U_{\text{axe}}(x, t)} = \frac{dF}{d\eta}$$

where

$$\eta = \frac{y}{l(x, t)}$$

The axial velocity U_{axe} and affinity function l are time dependent, whereas the dimensionless profile F is not.

The first problem $[t_p = o\ (t_j)]$ leads to a solution that is not physically interesting.[3] In the second case, we find the well-known affinity solutions[2] in which the jet Reynolds number has become a function of time t.

In the example, we have computed the solution for a periodic adiabatic compression with $\rho_{\max}/\rho_{\min} = A = 3$ at the compression half-period $t = T_P/2$ and a linear displacement of the pistons. The jet initial Reynolds number based on the nozzle thickness (or diameter) is $Re\,(0) = 63$. We notice in Fig. 2 a strong velocity profile variation more important in the case of the circular jet. The tendency is the same for the temperature profiles not represented here.

We see in Fig. 3 the important time dispersion of the emission lines of a fluid particle initially situated outside the jet.

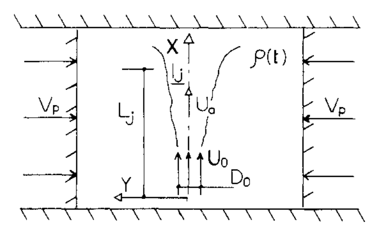

Fig. 1 Principle sketch of the phenomenon.

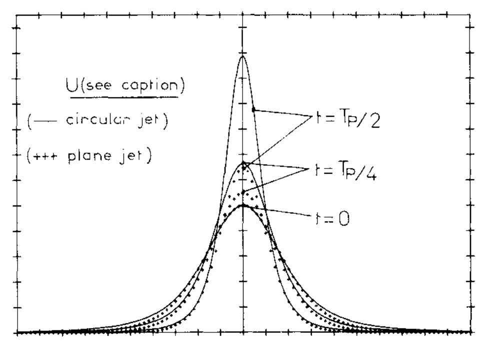

Fig. 2 Dimensionless velocity profile deformation of a laminar air jet for a linear displacement of the pistons with $A = 3$ for $t = 0$, $T_p/4$, $T_p/2$.

———— **circular jet** $\dfrac{U(x, r, t)}{U_{\text{axe}}(x, 0)}$ **vs** $\eta_0 = \dfrac{r}{l(x, 0)}$

+ + + + **plane jet** $\dfrac{U(x, y, t)}{U_{\text{axe}}(x, 0)}$ **vs** $\eta_0 = \dfrac{y}{l(x, 0)}$

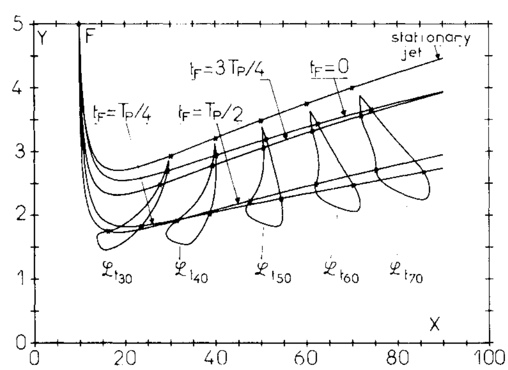

Fig. 3 Circular jet: trajectories of fluid particles passing through external point ($X_F = 10$, $Y_F = 5$) at $t_F = 0$, $T_P/4$, $T_P/2$, and $3T_P/4$. Rings $\mathcal{L}t_1$: position of the fluid particles at $t = t_F + t_1$ ($t_F \in [0, T_P]$; t_1 is fixed) for a linear displacement of the pistons with $A = 3$.

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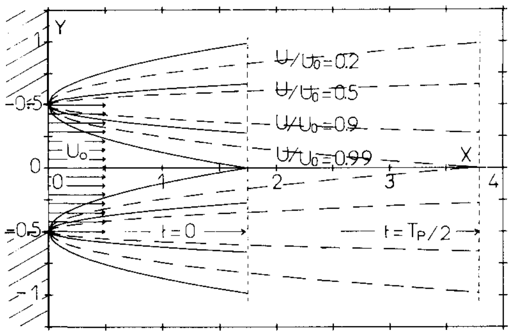

Fig. 4 Evolution of the mixing-layer isocurves $U/U_0 = 0.2, 0.5, 0.9,$ and 0.99 during the compression half-period for a linear displacement of the pistons with $A = 3$.

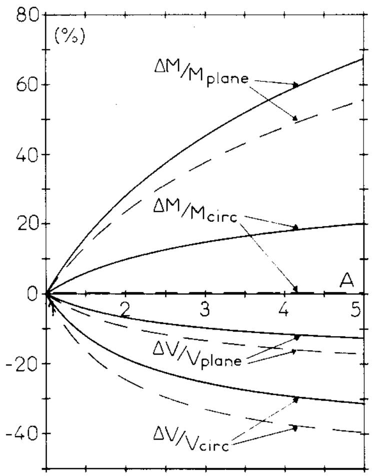

Fig. 5 $\Delta V/V$ (A) $[\Delta M(A)/M]$ Comparison of the fluid volume (mass) driven in both the excited jet and stationary jet over a compression period; ______ adiabatic compression; ---- isothermal compression.

II. Mixing Region and Potential Core

The development of the nozzle outflow has an influence on the space organization of the whole jet. The effect of the compression on the plane potential core is summarized in Fig. 4 ($\rho_{max}/\rho_{min} = 3$; Re (0) = 63). The affinity equation[2] has been solved numerically in order to obtain a good spatial localization of the mixing zone. The following relation can be derived:

$$L(t) = \frac{\nu(0)}{\nu(t)} L(0)$$

where $L(t)$ is the length of the potential core and $\nu(t)$ the kinematic viscosity, which shows that the lengthening is far from being negligible.

III. Discussion

To focus on the entrainment properties of the jet, we have introduced the quantities QM (x_1, x_2, t) $[QV$ $(x_1, x_2, t)]$ that measure the mass flux (the volume flux) of surrounding fluid driven in the jet between two cuts x_1 and x_2 along the jet axis $(x_1<x_2)$ at time t.

In Fig. 5, we compare the fluid volume and fluid mass driven in both an excited and stationary jet over a compression period for a compression rate A and a linear displacement of the pistons. We have

$$\frac{\Delta X}{X}(A) = \left[\frac{1}{T_P}\int_0^{T_P} \frac{QX\,(x_1, x_2, t)}{QX\,(x_1, x_2, 0)}\,\mathrm{d}t\right] - 1$$

where $X = V$ or $X = M$.

The positive value $(\Delta M/M)$ (A) represents the increase in the mass flow rate due to the global mass density rise. Nevertheless, in all cases and even for quite small values of A, we notice an important diminution of the volume mixed by the compressed jet.

We have summarized here some global properties of compressed laminar jet flows. A straightforward extrapolation to turbulent jets[2] will be possible if the behavior of compressed anisotropic turbulence can be modeled by a time-dependent turbulent viscosity. Further experimental investigation[3] is necessary.

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